Solutions to RSPL/3 (DS1)

1. (*b*) Radius of circle = 1 unit

Diameter of circle = 2 unit

Area of circle = $\pi(1)^2 = \pi$ sq. unit

Diameter of new circle = 4 cm

New radius = 2 cm

Area of new circle = $\pi(2)^2 = 4\pi$ sq unit

Increase in Area = $(4\pi - \pi) = 3\pi$ sq. unit

Increase in Area = $\frac{3\pi}{\pi} \times 100 = 300\%$

$\therefore$ Option (*b*) is correct.

2. (*d*) The quadratic polynomial is $x^2 - (p + 6)x + 2(2p + 1)$

Sum of zeros $(\alpha + \beta) = \frac{-[-(p+6)]}{1} = p + 6$

Product of zeros $\alpha\beta = \frac{2(2p+1)}{1} = 4p + 2$

According to question,

$$\Rightarrow \quad (p + 6) = \frac{1}{2}(4p + 2)$$
$$\Rightarrow \quad 2(p + 6) = 4p + 2$$
$$\Rightarrow \quad 2p + 12 = 4p + 2$$
$$\Rightarrow \quad 12 - 2 = 4p - 2p$$
$$\Rightarrow \quad 10 = 2p$$
$$p = 5$$

$\therefore$ Option (*d*) is correct.

3. (*c*) We have

$$y = x^3 - 16x$$
$$= x(x^2 - 16)$$
$$= x(x^2 - 4^2)$$
$$= x(x - 4)(x + 4)$$

For zeros of polynomial

$$\Rightarrow \quad x^3 - 16x = 0$$
$$\Rightarrow \quad x(x - 4)(x + 4) = 0$$
$$\Rightarrow \quad x = 0, x = 4, x = -4$$

$\therefore$ option (*c*) is correct.

4. (*a*) Let a be the first term and d be the common difference

here, $n = 63$

$$a_{32} = a + 31d = q \qquad \text{...(}i\text{)}$$

$$s_{63} = \frac{63}{2}[2a + (63 - 1)d]$$

$$= \frac{63}{2}[2a + 62d]$$

$$= \frac{63}{2} \times 2(a + 31d)$$

$$= 63(a + 31d)$$

$$= 63q \qquad \text{[from}(i)]$$

$\therefore \quad S_{63} = 63q$

$\therefore$ option (a) is correct.

5. (c) $$\frac{1-\cos\theta}{\sin\theta} + \frac{\sin\theta}{1-\cos\theta} = \frac{(1-\cos\theta)^2 + \sin^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{1+\cos^2\theta - 2\cos\theta + \sin^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{2(1-\cos\theta)}{(1-\cos\theta)\sin\theta} = \frac{2}{\sin\theta} = 2\text{cosec }\theta$$

$\therefore$ Option (c) is correct.

6. (d) The given quadratic equation is,

$$x^2 - 6x + 5 = 0$$

$$p + q = \frac{-(-6)}{1} = 6$$

$$pq = \frac{5}{1} = 5$$

$$p^3 + q^3 = (p + q)^3 - 3pq(p + q)$$

$$= 6^3 - 3 \times 5(6)$$

$$= 216 - 90$$

$$= 126$$

$\therefore$ Option (d) is correct.

7. (a) The given pair of linear equations are $2x - \frac{5}{2}y = 3$ and $-12x + my = n$

$$\frac{a_1}{a_2} = \frac{2}{-12}, \frac{b_1}{b_2} = \frac{\frac{-5}{2}}{m}, \frac{c_1}{c_2} = \frac{3}{n}$$

The given pair of lines are inconsistent

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \quad \frac{2}{-12} = \frac{\frac{-5}{2}}{m} \neq \frac{3}{n}$$

$$\therefore \quad \frac{2}{-12} \neq \frac{3}{n}$$

$$\therefore \quad \frac{1}{-6} \neq \frac{3}{n}$$

$$n \neq -18$$

$\therefore$ Option (a) is correct.

8. (*b*) $\tan\theta = \sqrt{3}$

$\Rightarrow \quad \tan\theta = \tan 60°$

$\Rightarrow \quad \theta = 60°$

$$\frac{\sec^2\theta - \text{cosec}^2\theta}{\sec^2\theta + \text{cosec}^2\theta} = \frac{\sec^2 60° - \text{cosec}^2 60°}{\sec^2 60° + \text{cosec}^2 60°} = \frac{(2)^2 - \left(\frac{2}{\sqrt{3}}\right)^2}{(2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= \frac{4 - \frac{4}{3}}{4 + \frac{4}{3}} = \frac{\frac{12-4}{3}}{\frac{12+4}{3}} = \frac{\frac{8}{3}}{\frac{16}{3}} = \frac{8}{3} \times \frac{3}{16} = \frac{1}{2}$$

$\therefore$ Option (*b*) is correct

9. (*c*) Let r_1 and r_2 be the radius of the two circles and R be radius of new circle

here $\quad r_1 = 5$ cm, $r_2 = 12$ cm,

$$\pi R^2 = \pi r_1^2 + \pi r_2^2$$

$$\pi R^2 = \pi[5^2 + 12^2]$$

$$R^2 = 25 + 144$$

$$R^2 = 169$$

$$R = 13 \text{ cm}$$

diameter = 2 radius = 2 × 13 = 26 cm

$\therefore$ option (*c*) is correct.

10. (*a*) Let a be the side of square

radius of incircle = $r_1 = \frac{a}{2}$

Diameter of incircle = a

Diameter of circumcircle = $\sqrt{2}a$

Radius of circumcircle = $r_2 = \frac{\sqrt{2}}{2}a$

$= r_2 = \frac{a}{\sqrt{2}}$

Now, $$\frac{\text{Area of incircle}}{\text{Area of circumcircle}} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{\pi\left(\frac{a}{2}\right)^2}{\pi\left(\frac{a}{\sqrt{2}}\right)^2}$$

$$= \frac{\frac{a^2}{4}}{\frac{a^2}{2}} = \frac{a^2}{4} \times \frac{2}{a^2} = \frac{1}{2}$$

$\therefore$ Ratio of area is 1 : 2

$\therefore$ Option (*a*) is correct.

11. Factors of 44 = 2 × 2 × 11 = $2^2 \times 11$

Factors of 121 = 11 × 11 = 11^2

HCF of 44 and 121 is 11

According to question,

$$\begin{aligned} 44m - 121 &= 11 \\ 44m &= 11 + 121 \\ 44m &= 132 \\ m &= \frac{132}{44} \\ \therefore \quad m &= 3 \end{aligned}$$

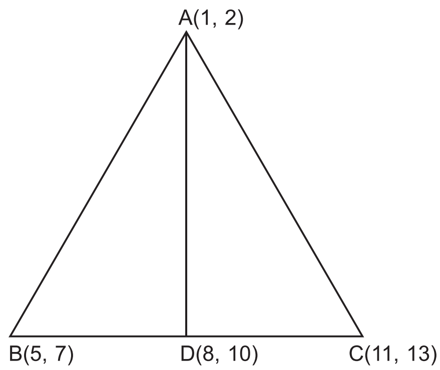
12. AD is the median from vertex A on the side BC

D is the mid-point of BC

$\therefore$ coordinates of $D\left(\frac{5+11}{2}, \frac{7+13}{2}\right)$

$\Rightarrow$ coordinates of D(8, 10)

$$\begin{aligned} |AD| &= \sqrt{(8-1)^2 + (10-2)^2} \\ &= \sqrt{7^2 + 8^2} \\ &= \sqrt{49 + 64} \\ &= \sqrt{113} \end{aligned}$$

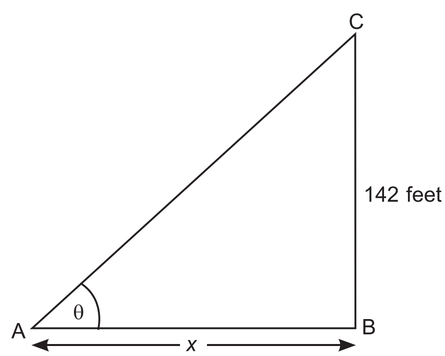


$\therefore$ length of median AD is $\sqrt{113}$

13. Let θ be sun's elevation, BC be the height and x be the shadow of tower.

Here θ = 60°, BC = 142 feet

$$\begin{aligned} \tan\theta &= \frac{BC}{AB} \\ \tan 60° &= \frac{142}{x} \\ \sqrt{3} &= \frac{142}{x} \\ x &= \frac{142}{\sqrt{3}} \end{aligned}$$



$\therefore$ Length of shadow $\frac{142}{\sqrt{3}}$ feet

OR

$$\begin{aligned} \operatorname{cosec}\theta - \cot\theta &= \frac{1}{3} \\ 3(\operatorname{cosec}\theta - \cot\theta) &= 1 \\ 3(\operatorname{cosec}\theta - \cot\theta) &= \operatorname{cosec}^2\theta - \cot^2\theta \\ 3(\operatorname{cosec}\theta - \cot\theta) &= (\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) \\ \operatorname{cosec}\theta + \cot\theta &= 3 \\ \therefore \quad \operatorname{cosec}\theta + \cot\theta &= 3 \end{aligned}$$

14. Total number of cards = 52

Number of red kings = 2

Probability of getting a red king = $\frac{2}{52} = \frac{1}{26}$

15. Let r be the radius of circle

$\Rightarrow \quad r = 1$ unit $\Rightarrow$ diameter = 2 unit

$\therefore$ Side of square = 2 unit

Area of square = side × side

= 2 × 2 = 4 sq. unit

16. $\angle QOR = 120°$, PQ and PR are tangents $OQ \perp PQ$ and $OR \perp PR$

[$\because$ tangents and radius are $\perp$ at the point of contact]

$\therefore \quad \angle OQP = 90°, \angle ORP = 90°$

In Quadrilateral QORP,

$\angle QOR + \angle ORP + \angle OQP + \angle QPR = 360°$

$120° + 90° + 90° + \angle QPR = 360°$

$\angle QPR = 360° - 300°$

$\therefore \quad \angle QPR = 60°$

17. Mean = 19.5, mode 36

By empirical relationship,

Mode = 3 median – 2Mean

$\Rightarrow$ 36 = 3 median – 2 × 19.5

$\Rightarrow$ 36 = 3 median – 39

$\Rightarrow$ 3 median = 36 + 39

3 median = 75

Median = $\frac{75}{3}$

Median = 25

OR

Mode is that observation which occurs maximum number of times,

Since if observation 18 and 20 are changed to 25, then item 25 occurs 11 times.

$\therefore$ mode is 11.

18. DE || AB if $\frac{AD}{CD} = \frac{BE}{CE}$

Now $\frac{3x+19}{x+3} = \frac{3x+4}{x}$

$\Rightarrow \quad x(3x + 19) = (x + 3)(3x + 4)$

$\Rightarrow \quad 3x^2 + 19x = 3x^2 + 4x + 9x + 12$

$\Rightarrow \quad 3x^2 + 19x = 3x^2 + 13x + 12$

$\Rightarrow \quad 3x^2 + 19x - 3x^2 - 13x = 12$

$\Rightarrow \quad 6x = 12$

$x = 2$

if $x = 2$, then DE || AB

19. Factors of 10 = 2 × 5

Factors of 15 = 3 × 5

Factors of 20 = 2 × 2 × 5 = 2^2 × 5

HCF of (10, 15, 20) = 5

LCM of (10, 15, 20) = 2^2 × 3 × 5 = 60

Ratio of HCF and LCM = $\frac{5}{60} = \frac{1}{12} \Rightarrow 1 : 12$

20. Let p be a rational number between $\sqrt{2}$ and $\sqrt{3}$.

$\therefore\ \sqrt{2} < p < \sqrt{3}$

On squaring throughout, we get

$2 < p^2 < 3$

The perfect squares which lies between 2 and 3 are, 2.25, 2.56, 2.89

We have,

$2 < 2.25 < 2.56 < 2.89 < 3$

Taking square root throughout,

$\sqrt{2} < 1.5 < 1.6 < 1.7 < \sqrt{3}$

$\therefore$ Rational numbers between $\sqrt{2}$ and $\sqrt{3}$ are 1.5, 1.6 and 1.7

21. Let a be the first term and d be the common difference of an AP.

here, $a = 7$

According to question,

$$s_4 = \frac{1}{2}[s_8 - s_4]$$

$$\Rightarrow \quad 2s_4 = s_8 - s_4$$

$$\Rightarrow \quad 2s_4 + s_4 = s_8$$

$$\Rightarrow \quad 3s_4 = s_8$$

$$3 \times \frac{4}{2}[(2 \times 7) + (4 - 1)d] = \frac{8}{2}[2 \times 7 + (8 - 1)d]$$

$$\Rightarrow \quad 6[14 + 3d] = 4(14 + 7d)$$

$$\Rightarrow \quad 84 + 18d = 56 + 28d$$

$$\Rightarrow \quad 28d - 18d = 84 - 56$$

$$\Rightarrow \quad 10d = 28$$

$$\Rightarrow \quad d = \frac{28}{10}$$

$$\Rightarrow \quad d = \frac{14}{5}$$

$$s_{30} = \frac{30}{2}\left[2 \times 7 + (30 - 1) \times \frac{14}{5}\right]$$

$$= 15\left[14 + 29 \times \frac{14}{5}\right]$$

$$= 15\left[\frac{70 + 406}{5}\right] = 3(476)$$

$$\therefore \quad s_{30} = 1428$$

OR

Let a be the first term and d be the common difference of an AP

According to question

$\Rightarrow \quad \frac{a_{11}}{a_{18}} = \frac{2}{3}$

$\Rightarrow \quad \frac{a+10d}{a+17d} = \frac{2}{3}$

$\Rightarrow \quad 3(a + 10d) = 2(a + 17d)$

$\Rightarrow \quad 3a + 30d = 2a + 34d$

$\Rightarrow \quad 3a - 2a = 34d - 30d$

$\Rightarrow \quad a = 4d$ ...(i)

So, $\quad \frac{s_5}{s_{10}} = \frac{\frac{5}{2}[2a+4d]}{\frac{10}{2}[2a+9d]} = \frac{1}{2}\left[\frac{2a+4d}{2a+9d}\right]$

Put $a = 4d$

$\Rightarrow \quad \frac{s_5}{s_{10}} = \frac{1}{2}\left[\frac{2\times 4d+4d}{2\times 4d+9d}\right] = \frac{1}{2}\left[\frac{12d}{17d}\right]$

$\therefore \quad \frac{s_5}{s_{10}} = \frac{6}{17}$

22. When a pair of dice is thrown,

Total number of elementary events = 36

Let A be the event of getting a doublets

Favourable elementary events are [(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)]

Favourable number of elementary events = 6

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

Let B be event that records a total of at least 10 when a pair of dice thrown by Vijay

Favourable elementary events are [(4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)]

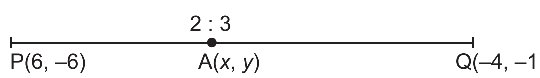
Total favourable outcomes = 6

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

Probability of Ajay and Vijay are equal

$\therefore$ Both of them have equal chances of winning.

23. Let co-ordinates of A be (x, y)



Given, $\quad \frac{PA}{PQ} = \frac{2}{5}$

$\Rightarrow \quad \frac{PA}{PA+AQ} = \frac{2}{5}$

$\Rightarrow \quad 5(PA) = 2(PA + AQ)$

$5PA - 2PA = 2AQ$

$3PA = 2AQ \Rightarrow \frac{PA}{AQ} = \frac{2}{3} = 2 : 3$

$$x = \frac{2 \times (-4) + 3 \times 6}{2+3}$$

$$x = \frac{-8+18}{5}$$

$$x = 2$$

$$y = \frac{2(-1)+3(-6)}{2+3}$$

$$y = \frac{-2-18}{5}$$

$$y = -4$$

$\therefore$ Coordinates of A(2, – 4)

24. $$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$$

Squaring both sides, we get

$$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$$

$$\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$$

$$\Rightarrow \quad \sin^2\theta = \cos^2\theta - 2\sin\theta\cos\theta$$

$$\Rightarrow \quad \cos^2\theta - 2\sin\theta\cos\theta = \sin^2\theta$$

Adding $\sin^2\theta$ on both sides

$$\Rightarrow \quad \cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta = \sin^2\theta + \sin^2\theta$$

$$\Rightarrow \quad (\cos\theta - \sin\theta)^2 = 2\sin^2\theta$$

$$\Rightarrow \quad (\cos\theta - \sin\theta) = \sqrt{2}\sin\theta$$

Hence proved.

OR

Here A = 15°

LHS = $4\sin 2A \cdot \cos 4A \cdot \sin 6A$

$= 4\sin(2 \times 15°) \cdot \cos(4 \times 15°) \cdot \sin(6 \times 15°)$

$= 4\sin 30° \cdot \cos 60° \cdot \sin 90°$

$= 4 \times \frac{1}{2} \times \frac{1}{2} \times 1 = 1 =$ RHS

Hence proved.

25. Let r be the radius of spherical marble

$\therefore$ diameter = $2r$ = 1.4 cm

$\Rightarrow$ r = 0.7 cm

Volume of spherical marble = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.7)^3 \text{ cm}^3 = \frac{1.372}{3}\pi \text{ cm}^3$

Let R be the radius and H be the height of cylindrical beaker increased

R = 7 cm, H = 28 cm

Volume of cylindrical beaker increased = $\pi R^2 H \text{ cm}^3$

$= \pi(7)^2 \cdot 28 = 1372\pi \text{ cm}^3$

$$\text{Number of marbles} = \frac{\text{Volume of cylindrical beaker}}{\text{Volume of each marble}}$$

$$= \frac{1372\pi}{\frac{1.372}{3}\pi} = \frac{1372\pi}{1.372\pi} \times 3 = 3000$$

26. Since, $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are two zeros of given polynomial $3x^4 + 6x^3 - 2x^2 - 10x - 5$

Then, $\left[x - \sqrt{\frac{5}{3}}\right]\left[x - \left(-\sqrt{\frac{5}{3}}\right)\right] = \left(x^2 - \frac{5}{3}\right)$ is a factor of given polynomial

$$
\begin{array}{r|l}
x^2 - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \quad (\; 3x^2 + 6x + 3 \\
 & \underline{\overset{-}{3x^4} \qquad\quad \overset{+}{-\,5x^2}} \\
 & \quad 6x^3 + 3x^2 - 10x - 5 \\
 & \quad \underline{\overset{-}{6x^3} \qquad\quad \overset{+}{-\,10x}} \\
 & \qquad 3x^2 - 5 \\
 & \qquad \underline{\overset{-}{3x^2} \overset{+}{-\,5}} \\
 & \qquad\quad 0
\end{array}
$$

For other zeros $3x^2 + 6x + 3 = 0$

$$
\begin{aligned}
3(x^2 + 2x + 1) &= 0 \\
x^2 + 2x + 1 &= 0 \\
x^2 + x + x + 1 &= 0 \\
x(x + 1) + 1(x + 1) &= 0 \\
(x + 1)(x + 1) &= 0 \\
x + 1 &= 0,\ x + 1 = 0 \\
x &= -1, \text{ or } x = -1
\end{aligned}
$$

$\therefore$ Other zeros are – 1 and – 1.

27. The given equations are

$$x + y = 5 \qquad ...(i)$$

and

$$x - y = 5 \qquad ...(ii)$$

Table for equation (*i*) is

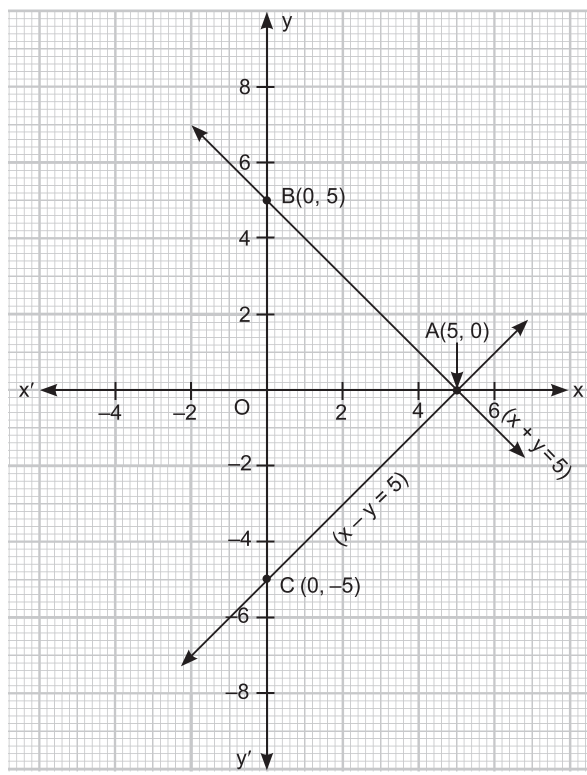
x	0	5	3
y	5	0	2

Table for equation (*ii*) is

x	0	5	3
y	–5	0	–2

The solution of the equation is (5, 0)

Coordinates of the triangle ABC are A(5, 0) B(0, 5), C(0, – 5)



$$
\begin{aligned}
\text{Area of } \Delta ABC &= \frac{1}{2} \times OA \times BC \\
&= \frac{1}{2} \times 5 \times 10 \\
&= 25 \text{ sq. units.}
\end{aligned}
$$

28. Let us assume that $2 + \sqrt{7}$ is not an irrational

$\therefore$ It can be written in the form $\frac{a}{b}$, where $b \neq 0$ and a and b are coprime.

So $$2 + \sqrt{7} = \frac{a}{b}$$

Subtracting 2 from both sides.

$$\sqrt{7} = \frac{a}{b} - 2$$

$$\sqrt{7} = \frac{a - 2b}{b}, \text{ this contradicts.}$$

LHS is an irrational number because $\sqrt{7}$ is an irrational number (given) and RHS is a rational number.

But, LHS is equal to RHS not possible.

Hence our supposition is wrong.

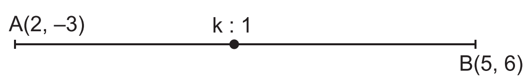
$\therefore$ $2 + \sqrt{7}$ is an irrational number.

29. Let required ratio be $k : 1$

The co-ordinates of point of division are $\left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$

This point lies on x-axis whose equation is $y = 0$,

$$\therefore \quad \frac{6k-3}{k+1} = 0$$



$$6k - 3 = 0$$

$$6k = 3$$

$$k = \frac{3}{6} = 1 : 2$$

$\therefore$ Co-ordinates are $\left(\frac{5 \times \frac{1}{2} + 2}{2+1}, \frac{6 \times \frac{1}{2} - 3}{2+1}\right) = \left(\frac{3}{2}, 0\right)$

OR

Here P(0, 3), Q(–2, y), R(–1, 4)

ΔPQR is a right angled triangle.

$$\therefore \quad RQ^2 = PQ^2 + PR^2$$

$$(-2 + 1)^2 + (y - 4)^2 = (-2 - 0)^2 + (y - 3)^2 + (-1 - 0)^2 + (4 - 3)^2$$

$$\Rightarrow \quad 1 + y^2 + 16 - 8y = 4 + y^2 + 9 - 6y + 1 + 1$$

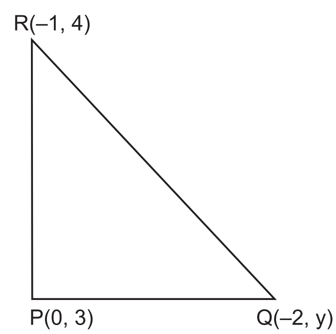
$$\Rightarrow \quad y^2 + 17 - 8y = y^2 + 15 - 6y$$

$$\Rightarrow \quad 17 - 8y = 15 - 6y$$

$$\Rightarrow \quad 17 - 15 = -6y + 8y$$

$$\Rightarrow \quad 2 = 2y$$

$$\Rightarrow \quad y = 1$$



$\therefore$ co-ordinates of Q(–2, 1)

$$\text{Area of } \Delta PQR = \frac{1}{2}|0[1-4]+(-2)(4-3)+(-1)(3-1)|$$
$$= \frac{1}{2}|0-2-2|$$
$$= \frac{1}{2}|-4|$$
$$= \frac{1}{2}\times 4 = 2 \text{ sq. units}$$

30. Let marks secured by Jitesh is x and marks secured by Raman is y

Such that $x > y$.

According to question

$$x^2 - y^2 = 45 \quad ...(i)$$

and $$y^2 = 4x \quad ...(ii)$$

from (*i*) and (*ii*)

$$x^2 - 4x = 45$$
$$x^2 - 4x - 45 = 0$$
$$x^2 - 9x + 5x - 45 = 0$$
$$x(x-9) + 5(x-9) = 0$$
$$(x-9)(x+5) = 0$$
$$x - 9 = 0 \text{ or } x + 5 = 0$$
$$x = 9 \text{ or } x = -5$$

Rejecting $x = -5$

$\therefore$ $x = 9$

Substituting $x = 9$ in (*ii*)

$$y^2 = 4 \times 9$$
$$y^2 = 36$$
$$y = \pm 6 \qquad \text{Rejecting } x = -6$$

$\therefore$ Marks secured by Jitesh is 9

Marks secured by Raman is 6

OR

$$\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5}$$
$$\frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$$

Let $\frac{1}{3x+2y} = A$, $\frac{1}{3x-2y} = B$

$$\therefore \quad 2A + 3B = \frac{17}{5} \quad ...(i)$$
$$5A + B = 2 \quad ...(ii)$$

Multiplying equation (*ii*) by 3 and subtract it from (*i*) we get

$$2A + 3B = \frac{17}{5} \quad \text{...}(i)$$

$$\underset{-}{15A} \underset{-}{+} 3B = \underset{-}{6} \quad \text{...}(iii)$$

$$-13A = \frac{-13}{5}$$

$$\Rightarrow \quad A = \frac{1}{5}$$

Substituting in (*ii*) we get

$$5 \times \frac{1}{5} + B = 2$$

$$1 + B = 2$$

$$B = 1$$

$$\frac{1}{3x + 2y} = \frac{1}{5} \quad \text{and} \quad \frac{1}{3x - 2y} = 1$$

$$3x + 2y = 5 \quad \text{...}(iv) \quad \text{and} \quad 3x - 2y = 1 \quad \text{...}(v)$$

Adding (*iv*) and (*v*)

$$3x + 2y = 5$$

$$3x - 2y = 1$$

$$6x = 6$$

$$\Rightarrow \quad x = 1$$

Substituting in (*iv*)

$$3 \times 1 + 2y = 5$$

$$2y = 5 - 3$$

$$2y = 2$$

$$y = 1$$

Hence $x = 1, y = 1$ are the solution of the given equations.

31. Given $a \cos\theta - b \sin\theta = x$

and $a \sin \theta + b \cos \theta = y$

Squaring both sides of the equations and adding

$$(a \cos \theta - b \sin \theta)^2 + (a \sin \theta + b \cos \theta)^2 = x^2 + y^2$$

$$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta - 2ab \sin \theta \cos \theta + a^2 \sin^2\theta + b^2\cos^2\theta + 2ab \sin\theta\cos\theta = x^2 + y^2$$

$$\Rightarrow a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2 \theta + \sin^2\theta) = x^2 + y^2$$

$$\Rightarrow \quad a^2 \times 1 + b^2 \times 1 = x^2 + y^2 \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow \quad a^2 + b^2 = x^2 + y^2$$

Or

LHS $\dfrac{\tan^3\theta}{1+\tan^2\theta}+\dfrac{\cot^3\theta}{1+\cot^2\theta} = \dfrac{\tan^3\theta}{\sec^2\theta}+\dfrac{\cot^3\theta}{\text{cosec}^2\theta}$

$$= \dfrac{\dfrac{\sin^3\theta}{\cos^3\theta}}{\dfrac{1}{\cos^2\theta}}+\dfrac{\dfrac{\cos^3\theta}{\sin^3\theta}}{\dfrac{1}{\sin^2\theta}}$$

$$= \frac{\sin^3\theta}{\cos^3\theta}\times\frac{\cos^2\theta}{1}+\frac{\cos^3\theta}{\sin^3\theta}\times\frac{\sin^2\theta}{1}$$

$$= \frac{\sin^3\theta}{\cos\theta}+\frac{\cos^3\theta}{\sin\theta}$$

$$= \frac{\sin^4\theta+\cos^4\theta}{\sin\theta\cos\theta}$$

$$= \frac{(\sin^2\theta)^2+(\cos^2\theta)^2}{\sin\theta\cos\theta}$$

$$= \frac{(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cdot\cos\theta}$$

$$= \frac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cdot\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}-\frac{2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$= \text{cosec}\,\theta\sec\theta - 2\sin\theta\cos\theta$

= RHS Hence proved

32. (*i*) More than ogive: Downward sloping curve is more than ogive.

Less than ogive: Upward sloping curve is less than ogive.

(*ii*) Median is the x-coordinate of point of intersection of less than ogive and more than ogive. The curves intersect each other at point P(30, 25) x-coordinate is 30.

$\therefore$ Median is 30

(*iii*) Median 30, mode = 37.85

By Empirical relationship.

Mode = 3 median – 2 mean

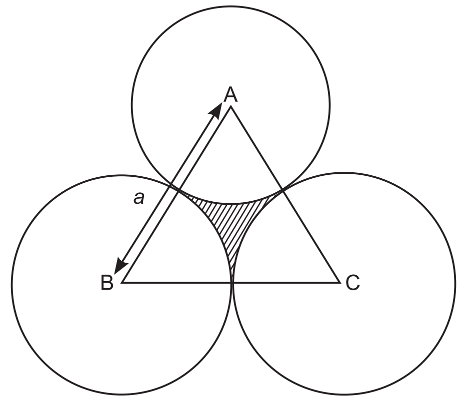
37.85 = 3 × 30 – 2 mean

$\Rightarrow$ 2 mean = 90 – 37.85

Mean $= \dfrac{52.15}{2}$

$\therefore$ Mean = 26.075

33. Let the side of equilateral triangle be a cm



$$\text{Area of equilateral } \Delta = \frac{\sqrt{3}}{4} \times a^2$$

$$\Rightarrow \quad 49\sqrt{3} = \frac{\sqrt{3}}{4}a^2$$

$$\Rightarrow \quad a^2 = 196$$

$$\Rightarrow \quad a = 14 \text{ cm}$$

$$\text{Radius of circle} = \frac{1}{2}(\text{side of } \Delta)$$

$\therefore$ Radius of circle = 7 cm

Area of 3 sectors of a circle with central angle 60°

$$= 3\frac{\pi r^2 \theta}{360°}$$

$$= 3 \times \frac{22}{7} \times 7 \times 7 \times \frac{60}{360} = 77 \text{ cm}^2$$

Area of triangle not included in circle = Area of equilateral Δ – Area of 3 sectors

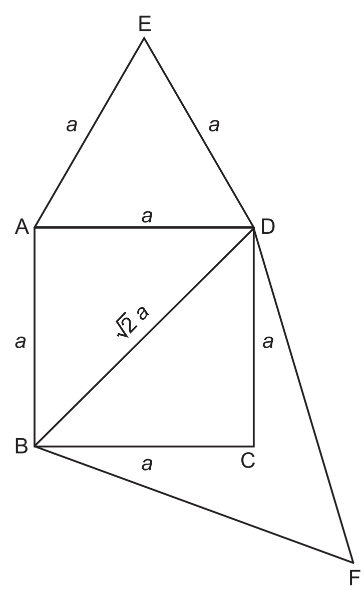
$$= (49\sqrt{3} - 77) \text{ cm}^2$$

$$= (49 \times 1.73 - 77) \text{ cm}^2$$

$$= (84.77 - 77) \text{ cm}^2$$

$$= 7.77 \text{ cm}^2$$

34. Given: Let the equilateral triangle formed on one side of square is ΔAED and another equilateral triangle formed on its diagonal BD is ΔBDF.



To prove: $\text{ar}(\Delta AED) = \frac{1}{2}\text{ ar}(\Delta BFD)$

Proof: Area of equilateral $\Delta AED = \frac{\sqrt{3}}{4} \times (\text{side})^2$

Let the side of square ABCD be a units

$$= \frac{\sqrt{3}}{4} \times a^2 \text{sq. unit} \quad ...(i)$$

Diagonal of square BD = $\sqrt{2}$ side = $\sqrt{2}a$ unit

Area of equilateral triangle $\Delta BFD = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$$= \frac{\sqrt{3}}{4} \times (\sqrt{2}a)^2 = \frac{\sqrt{3}}{4} \times 2a^2$$

$$= \frac{\sqrt{3}}{2}a^2 \quad ...(ii)$$

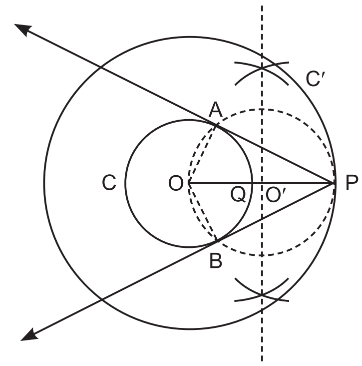
Equating (i) and (ii), we get

$$\Rightarrow \quad \text{ar } \Delta BFD = 2 \times \text{ar}(\Delta AED)$$

$$\therefore \quad \text{ar of } \Delta AED = \frac{1}{2}\text{ar } (\Delta BFD)$$

Hence proved.

35. Steps of construction:



(*i*) Draw concentric circles of radii OQ = 2 cm and OP = 5cm having same centre O.

(*ii*) Mark these circles as C and C′.

(*iii*) O, Q and P lie on the same line.

(*iv*) Draw perpendicular bisector of OP which intersect OP at O′.

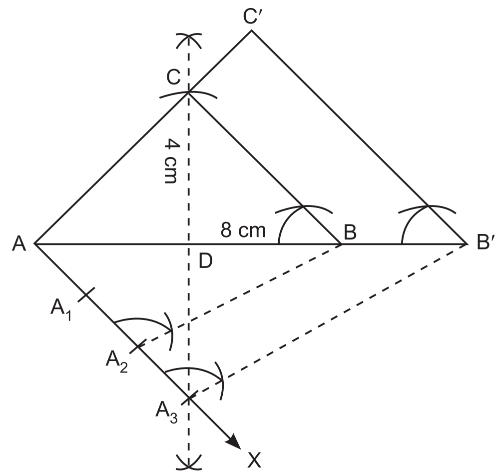
(*v*) Take O′ as centre, draw a circle of radius OO′ which intersect circle C at point A and B.

(*vi*) Join PA and PB, these are the required tangents.

(*vii*) Lengths of these tangents are approximately 4.6 cm

OR

Steps of construction:



(*i*) Draw base AB = 8 cm

(*ii*) Draw perpendicular bisector of AB. Mark CD = 4 cm, on ⊥ bisector, where D is the mid point of AB.

(*iii*) Draw an acute angle BAX, below AB at point A.

(*iv*) Mark the ray AX with A_1, A_2, A_3, such that $AA_1 = A_1A_2 = A_2A_3$

(*v*) Join A_2 to B. Draw $A_3B' \mid\mid A_2B$, where B′ is a point on extended line AB.

(*vi*) At B′, draw $B'C' \mid\mid BC$, where C′ is point on extended line AC.

(*vii*) $\Delta AB'C'$ is the required triangle.

36. Let total number of rows is x and number of students in each row be y.

$\therefore$ Total students in the class = xy

According to question

$$(x-1)(y+3) = xy$$
$$xy + 3x - y - 3 = xy$$
$$3x - y = 3 \quad ...(i)$$

According to question,

$$(x+2)(y-3) = xy$$
$$xy - 3x + 2y - 6 = xy$$
$$-3x + 2y = 6 \quad ...(ii)$$

Multiplying equation (*i*) by 2,

$\Rightarrow \quad 6x - 2y = 6 \quad ...(iii)$

Adding (*ii*) and (*iii*),

$$-3x + 2y + 6x - 2y = 6 + 6$$
$$3x = 12$$
$$x = 4$$

Substitute in (*i*), we get

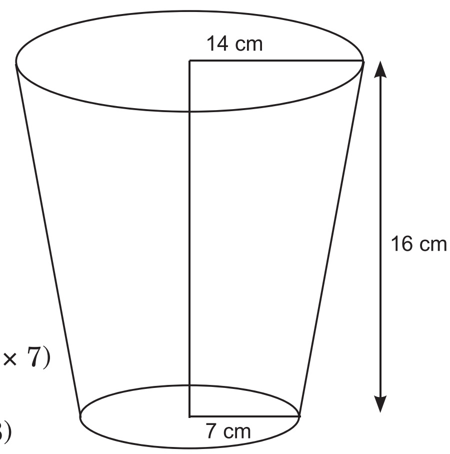
$$3(4) - y = 3$$
$$12 - 3 = y$$
$$9 = y$$

$\therefore$ Number of rows = 4

Number of students in each row = 9

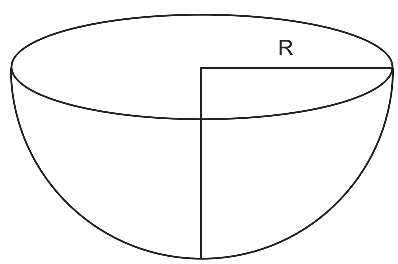
Total students in the class = $xy = 4 \times 9 = 36$

37. For bucket,



$$h = 16 \text{ cm}$$
$$r_1 = 14 \text{ cm}$$
$$r_2 = 7 \text{ cm}$$
$$V_1 = \text{Volume of Bucket}$$
$$= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$
$$V_1 = \frac{1}{3}\pi(16)(14^2 + 7^2 + 14 \times 7)$$
$$= \frac{1}{3}\pi(16)(196 + 49 + 98)$$
$$= \frac{1}{3} \times \pi(16) \times (343)$$
$$V_1 = \frac{5488}{3}\pi \text{ cm}^3$$

Let R be the radius of hemispherical bowl



V_2 = Volume of hemisphere = $\frac{2}{3}\pi R^3$

$V_1 = V_2$

$$\Rightarrow \quad \frac{5488}{3}\pi = \frac{2}{3}\pi R^3$$
$$\Rightarrow \quad R^3 = \frac{5488}{3} \times \frac{3}{2}$$
$$R^3 = 2744$$

$$R^3 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$$

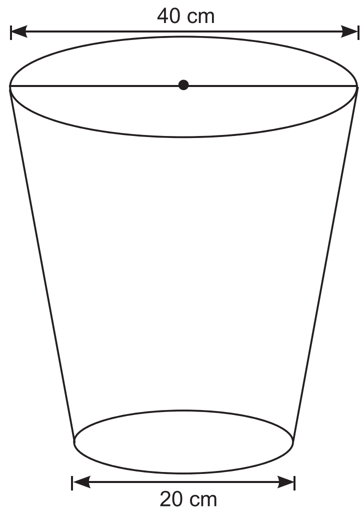
$$R^3 = (14)^3$$

$$R = 14 \text{ cm}$$

Internal diameter of hemisphere = 2R = 2 × 14 = 28 cm

OR

Here $V = 17600 \text{ cm}^3$



$$r_1 = \frac{40}{2} = 20 \text{ cm}$$

$$r_2 = \frac{20}{2} = 10 \text{ cm}$$

Let h be the height of frustum

$$\text{Volume (V)} = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{1}{3}\pi h(20^2 + 10^2 + 20 \times 10)$$

$$= 17600 \text{ cm}^3$$

$$\therefore \quad \frac{1}{3} \times \frac{22}{7} h\,(400 + 100 + 200) = 17600$$

$$\therefore \quad \frac{1}{3} \times \frac{22}{7} \times h \times 700 = 17600$$

$$h = \frac{17600 \times 3 \times 7}{22 \times 700}$$

$$h = 24 \text{ cm}$$

$$\text{Slant height of bucket} = l = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$= \sqrt{24^2 + (20 - 10)^2}$$

$$= \sqrt{576 + 100} = \sqrt{676}$$

$$\therefore \quad l = 26 \text{ cm}$$

$$\text{Total surface area of bucket} = \pi[(r_1 + r_2)l + r_2^2]$$

$$= \frac{22}{7}[(20 + 10)26 + (10)^2]$$

$$= \frac{22}{7}(780 + 100)$$

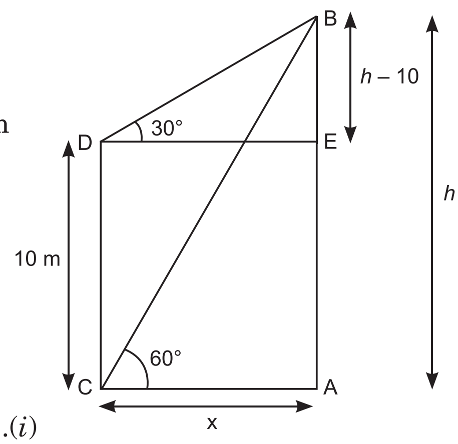
$$= \frac{22}{7} \times 880 = 2766 \text{ cm}^2 \text{ (Approx)}$$

38. Let AB be the tower of height h m

Let CA = x m,

CA = DE = x

$$BE = AB - AE = (h - 10)\,m$$



CD = 10 m

In rt. ΔCAB

$$\tan 60° = \frac{AB}{AC}$$

$$\sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \quad ...(i)$$

In rt. ΔDEB

$$\tan 30° = \frac{BE}{DE}$$

$$\frac{1}{\sqrt{3}} = \frac{h-10}{x} \quad [\because \ AC = DE = x]$$

$$\Rightarrow \quad x = \sqrt{3}(h-10)$$

$$\frac{h}{\sqrt{3}} = \sqrt{3}(h-10) \quad \text{[From } (i)]$$

$$h = 3(h-10)$$

$$h = 3h - 30$$

$$2h = 30$$

$$h = 15$$

∴ Height of tower is 15 m.

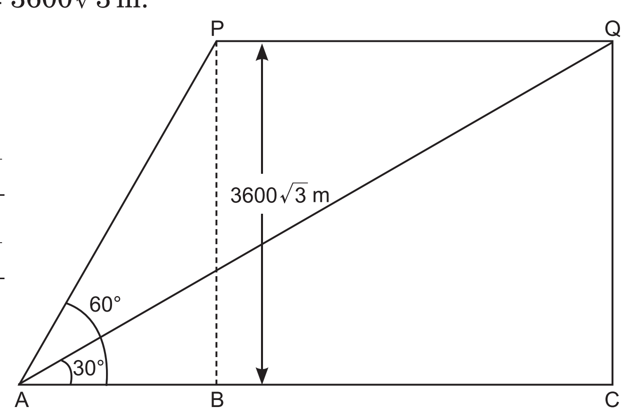
OR

Let P and Q be the two positions of the plane and let A be the point of observation. Let ABC be horizontal line through A.

It is given that angles of elevation of the plane in two positions P and Q from point A are 60° and 30° respectively.

∴ ∠PAB = 60°, ∠QAB = 30° and PB = $3600\sqrt{3}$ m.

In ΔABP



$$\tan 60° = \frac{BP}{AB}$$

$$\sqrt{3} = \frac{3600\sqrt{3}}{AB}$$

$$AB = \frac{3600\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \quad AB = 3600 \text{ m}$$

In ΔACQ, we have

$$\tan 30^\circ = \frac{CQ}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{AC}$$

$$AC = 3600 \times 3$$

$$AC = 10800 \text{ cm}$$

$\therefore$ PQ = BC = AC – AB = 10800 – 3600 = 7200 m

Thus, the plane travel 7200 m in 30 seconds.

Hence, speed of plane = $\frac{7200}{30}$ = 240 m/s

$$\Rightarrow \quad \frac{240}{1000} \times 3600 \text{ km/hr} = 864 \text{ km/hr}$$

39.

C.I.	Frequency	Cumulative frequency	
10 – 20	12	12	
20 – 30	30	42	
30 – 40	f_1	$42 + f_1$	
40 – 50	65	$107 + f_1$	→ Median Class
50 – 60	f_2	$107 + f_1 + f_2$	
60 – 70	25	$132 + f_1 + f_2$	
	18	$150 + f_1 + f_2$	
Total	230		

Median = 46, N = 230

Median class is 40 – 50

Here $l = 40, f = 65, c.f. = 42 + f_1, h = 10$

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$46 = 40 + \frac{\frac{230}{2} - (42 + f_1)}{65} \times 10$$

$$6 = \frac{(230 - 84 - 2f_i)}{130} \times 10$$

$$78 = 146 - 2f_1$$

$$2f_1 = 146 - 78$$

$$2f_1 = 68$$

$$f_1 = 34$$

Now $\quad 150 + f_1 + f_2 = 230$

$$f_1 + f_2 = 230 - 150 = 80$$

$$34 + f_2 = 80$$
$$f_2 = 80 - 34 = 46$$
$\therefore \quad f_1 = 34$ and $f_2 = 46$

40. **Given:** AB and AC are two tangents drawn from an external point to the circle with centre O.

To prove: AB = AC

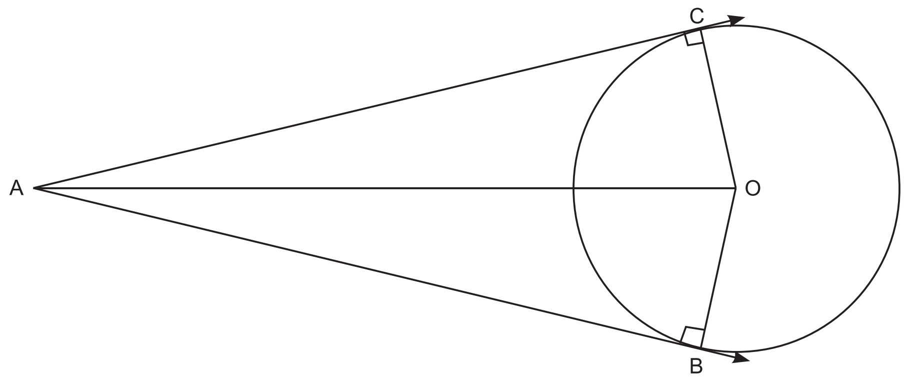
Construction: Join OA

Proof: In ΔAOB and ΔAOC

$\angle ABO = \angle ACO = 90°$

[$\because$ radius of circle is perpendicular to the tangent at the point of contact]

$AO = AO$ (common)



$OB = OC$ (radii of circle)

$\therefore \quad \Delta AOB \cong \Delta AOC$

$\therefore \quad AB = AC$ (cpct)

Hence, length of tangents from on external point are equal.

Other part:

Given: GC = 3 cm, BC = 7 cm, AH = 6 cm

$GC = CF$
$AH = AE$
$BF = BE$
[Lengths of tangents from an external point are equal.]

$GC = CF = 3cm$
$BC = 7$ cm
$BC = BF + CF = 7$ cm
$BF + 3 = 7$
$BF = 7 - 3$
$BF = 4$ cm
$BF = BE = 4$ cm
$AH = AE = 6$ cm
$AB = AE + BE$
$= 4 + 6$ cm $= 10$ cm